DoA Estimation with Compensation of Hardware Impairments

Daniele Inserra and Andrea M. Tonello
DIEGM - Università di Udine - via delle Scienze 208 - 33100 Udine - Italy
phone: +39 0432 558042 - fax: +39 0432 558251 - e-mail: daniele.inserra@uniud.it, tonello@uniud.it

Abstract—We consider the estimation of the direction of arrival (DoA) in the presence of hardware impairments that include the RF carrier frequency offsets, the phase offsets generated by an uncalibrated array, and the DC offset. The impairments model has been derived from the analysis of a hardware multiple antenna test-bed that uses direct down-conversion RF front-ends. The performance of the proposed algorithm has been investigated both analytically and via simulations. We show that the estimator provides good performance for a wide range of angles and it is robust to the hardware impairments.

I. INTRODUCTION

The estimation of a signal direction of arrival (DoA) has attracted considerable research efforts. Several algorithms have been proposed and they exploit the phase difference of the signals received by distinct antenna elements [1]-[2]. The performance of the estimator can be severely affected by the presence of hardware impairments. Some of these impairments have been considered in the literature among which the array uncalibration that causes a phase ambiguity [3]-[4], the imperfect knowledge of the array position [5], the mutual coupling of the array elements [6]. However, there are also other impairments that need to be taken into account as the presence of offsets among the emitter and the receiver carrier frequency, and the presence of a DC bias. Carrier frequency offsets can also be introduced because of the emitter movement in vehicular scenarios.

In this paper we consider a system model that takes into account the presence of an uncalibrated array, carrier and phase offsets among the transceivers, as well as the presence of a DC bias. The model has been derived by the experimental characterization of a hardware test-bed that comprises a four element linearly spaced array, four direct-conversion RF receivers, and an eight channel acquisition board with FPGA. The test-bed operates at 2.4 GHz and it is based on the Lyrtech platform [7]. We have not considered the I&Q imbalance problem because measurements have revealed that in our test-bed it is of negligible entity.

The base-band model is described in Section II. An algorithm for the compensation of the impairments and the DoA estimation is presented in Section III. The algorithm mitigates the DC offsets via filtering the signals down converted at an intermediate low frequency. Then, a spatio-temporal differential metric is implemented to cancel the time variant phase ambiguity generated by the carrier frequency offsets. We study the performance of the estimator in Section IV. In particular, we evaluate the Cramer-Rao bound. Simulation results are reported in Section VI.

II. SYSTEM MODEL DESCRIPTION

We consider a source that emits a single tone signal $s(t) = e^{j2\pi f_0,RF t}$ and a receiver equipped with a linearly equispaced antenna array of $M$ elements. Assuming a plane waves line-of-sight propagation scenario, the incident signal at the $i$-th antenna element (sensor) can be written as

$$x_{RF}^{(i)}(t) = \rho^{(i)} e^{j2\pi f_0,RF(t-\Delta t^{(i)})} + w_{RF}^{(i)}(t), \ i \in [1, M]$$

where $f_{0,RF}$ is the plane wave frequency, $\tau$ is the propagation delay between the emitter and the receiver array, $\Delta t^{(i)}$ is the propagation delay between the first sensor and the $i$-th one caused by a non-zero propagation angle $\phi$ between the incident signal and the array (Fig. 1), and $w_{RF}^{(i)}(t)$ is the additive noise. We also assume the attenuation $\rho^{(i)}$ (propagation loss) to be time invariant during the DoA estimation.

Under the plane wave propagation assumption, we can express the propagation delay as

$$\Delta t^{(i)} = \frac{d}{c_0} (i-1) \sin(\phi), \ i \in [1, M]$$

where $d$ is the distance between the sensors and $c_0$ is the speed of light.

We consider a direct-conversion RF hardware architecture as sketched in Fig. 1 followed by sampling the signals with period $T$. Assuming to down-convert the RF signals to a low frequency $f_0$, the sequence of samples for the $i$-th sensor can be written as

$$x^{(i)}(nT) = x_{n}^{(i)}$$

$$= \alpha^{(i)} e^{j(\psi_n^{(i)} - \Phi^{(i)})} + w_{n}^{(i)}, \ i \in [1, M]$$

where

$$\psi_n^{(i)} = 2\pi(f_0 + \Delta f^{(i)}) n T - 2\pi f_{0,RF} \tau - 2\pi f_{0,RF} \Delta t^{(i)}$$

$$f_{c}^{(i)} = f_{0,RF} - f_0 - \Delta f^{(i)}$$

is the down-conversion carrier frequency of the $i$-th receiver. We assume it to be somewhat smaller than $f_{0,RF}$ to allow the elimination, as it is explained in the following, of the DC offset $w_{DC}^{(i)}$, that is introduced by the direct conversion chain. Furthermore, each receiver suffers of a carrier frequency offset $\Delta f^{(i)}$, and a phase offset $\Phi^{(i)}$. Also, we assume $w_{n}^{(i)}$ circularly symmetric Gaussian, with mean $w_{DC}^{(i)}$ and variance per component $N_0/2$. Finally, $\alpha^{(i)}$ is...
the time-invariant channel gain that includes the propagation loss and the receiver gain. It is real due to the fact that we neglect the I&Q imbalance. Furthermore, we denote the carrier phase offsets with $\Phi^{(i)}$.

The estimation of the DoA can be done exploiting the phase difference among the sensors $2\pi f_0,RF \Delta f^{(i)}$ that is due to the different propagation delay. Unfortunately, the presence of the hardware impairments causes a phase uncertainty as a result of the presence of the contribution from the carrier frequency offset $2\pi(f_0 + \Delta f^{(i)})NT$ and the carrier phase offset $\Phi^{(i)}$. Furthermore, the DC offset is also a source of error.

III. JOINT IMPAIRMENT COMPENSATION AND DOA ESTIMATION

In this section we describe the proposed algorithm for DoA estimation and the compensation of the hardware impairments. It is based on first removing the DC offset. Then, we perform a spatio-temporal differential operation.

A. Impairment Compensation

To remove the DC offset, we down-convert the RF signals to the intermediate low frequency $f_0$ and we sample them to obtain (3). Then, we implement a digital band-pass filter to obtain

$$x^{(i)}_{F,n} = h \ast x^{(i)}_{n} = \alpha^{(i)}e^{j\psi^{(i)} - \Phi^{(i)}} + w^{(i)}_{F,n}, \quad i \in [1, M]$$

where we have assumed the carrier frequency offsets to be small compared to the filter bandwidth. The filtered noise $w^{(i)}_{F,n}$ has approximately zero mean depending on the filter selectivity. In the following analysis we assume that the filtering totally removes the DC offset effect, so that the noise $w^{(i)}_{F,n}$ has zero mean.

Now, if we assume the carrier frequency offsets of the $M$ receivers identical, i.e., $\Delta f^{(i)} = \Delta f, \forall i \in [1, M]$, we can perform a differential operation among pair of distinct antenna signals to remove them. This yields the signals

$$z^{(i)}_n = x^{(2i-1)}_{F,n}, \quad i \in [1, M/2]$$

where

$$\tilde{\psi} = 2\pi f_0,RF \frac{d}{c_0} \sin(\phi)$$

$$\hat{\alpha}^{(i)} = \alpha^{(2i-1)} \cdot \alpha^{(2i)}$$

and the noise contribution is

$$\tilde{w}^{(i)}_n = w^{(2i-1)}_{F,n} \cdot \alpha^{(2i)} + w^{(2i-1)}_{F,n} \alpha^{(2i)} e^{-j(\tilde{\psi} - \Phi^{(2i)})} + w^{(2i)}_{F,n} \alpha^{(2i)} e^{j(\tilde{\psi} - \Phi^{(2i)})}$$

As (6) reveals, the time variant phase ambiguity introduced by the carrier frequency offset is removed, and we are left with the useful phase $\tilde{\psi}$ that is a function of the angle of arrival $\phi$. Furthermore, the noise contribution has zero mean under the assumption of $w^{(2i-1)}_{F,n}$ and $w^{(2i)}_{F,n}$ to be independent with zero mean, and it has variance

$$\sigma^2_{\tilde{w}^{(i)}} = |\alpha^{(2i)}|^2 \sigma^2_{w^{(2i-1)}_{F,n}} + |\alpha^{(2i-1)}| \sigma^2_{w^{(2i)}_{F,n}} + \sigma^2_{\tilde{w}^{(2i-1)}_{F,n}} \sigma^2_{\tilde{w}^{(2i)}_{F,n}}$$

where $\sigma^2_{\tilde{w}^{(i)}}$ is the variance of $\tilde{w}^{(i)}_n$. There is still a phase ambiguity $\hat{\Phi}^{(i)}$ that needs to be removed from the useful signal. This is due to the array uncalibration and the receiver carrier phase offsets. A method to remove it consists in averaging (in space) the antenna signals as proposed in [3] under the assumption that the receiver phases are independent with zero mean so that $\sum_{i=1}^{M/2} \hat{\Phi}^{(i)} = 0$. This is a reasonable approach if the number of sensors is large. Another method uses a pre-calibration step. This can be achieved either using a local reference signal or performing a DoA pre-estimation with a transmitting node at a known position. With this last method the signals (6) assuming $\hat{\Phi}_{REF} = 0$ read

$$z^{(i)}_{REF,n} = \hat{\alpha}^{(i)}e^{-j\hat{\Phi}^{(i)}} + \tilde{w}^{(i)}_{REF,n}, \quad i \in [1, M/2]$$

where $\tilde{w}^{(i)}_{REF,n}$ has variance $\sigma^2_{\tilde{w}^{(i)}}$. We proceed by implementing another differential operation between (6) and (10) with a lag of $N$ samples to obtain

$$\hat{z}^{(i)}_n = z^{(i)}_{n+N} - z^{(i)}_{REF,n} = |\hat{\alpha}^{(i)}|^2 e^{j\tilde{\psi}} \tilde{w}^{(i)}_n, \quad i \in [1, M/2]$$

where the noise term

$$\tilde{w}^{(i)}_n = \tilde{w}^{(i)}_{F,n} \alpha^{(i)} e^{-j(\tilde{\psi} - \Phi^{(i)})} + \tilde{w}^{(i)}_{n+N} \alpha^{(i)} e^{-j(\tilde{\psi} - \Phi^{(i)})}$$

and we sample them to obtain (3). Then, we implement a digital band-pass filter to

$$x^{(i)}_{F,n} = h \ast x^{(i)}_{n} = \alpha^{(i)}e^{j\psi^{(i)} - \Phi^{(i)}} + w^{(i)}_{F,n}, \quad i \in [1, M]$$

where we have assumed the carrier frequency offsets to be small compared to the filter bandwidth. The filtered noise $w^{(i)}_{F,n}$ has approximately zero mean depending on the filter selectivity. In the following analysis we assume that the
B. DoA Estimation

The DoA can be obtained with a least square approach from (11) as follows

\[ z = \frac{2}{NM} \sum_{i=1}^{M/2} \sum_{n=0}^{N-1} \hat{z}_{n}^{(i)} = Ae^{j\psi} + W \]  

(13)

where

\[ A = \frac{2}{M} \sum_{i=1}^{M/2} |\hat{\alpha}^{(i)}|^2, \quad W = \frac{2}{NM} \sum_{i=1}^{M/2} \sum_{n=0}^{N-1} \hat{w}_{n}^{(i)}. \]  

(14)

Finally, the angle of arrival \( \phi \) is obtained as follows

\[ \hat{\phi} = \arcsin \left( \frac{\cos \Delta t}{d} \right) \]  

(15)

\[ \hat{\Delta} t = \frac{\Delta z}{2\pi f_{0,RF}} \]  

(16)

where \( \cdot \) is the argument operator.

In summary, the estimation of the angle of arrival in the presence of the hardware impairments comprises the following steps:

- we down-convert the array RF signals to an intermediate low frequency \( f_0 \), we sample and filter them with a band-pass filter to remove the DC offset;
- we perform a differential operation among pair of antenna signals to remove the carrier frequency offsets;
- we perform the previous operation first with a known position emitter to obtain a reference array calibration signal;
- we perform another differential operation among the reference signal and the unknown direction signal to remove the phase ambiguity mainly due to an uncalibrated array;
- we perform time and space averaging according to (13) and finally compute the angle-of-arrival.

IV. PERFORMANCE ANALYSIS

In this section we study the mean square error of the estimator that is defined as

\[ MSE(\phi) = E\{ |\phi - \hat{\phi}|^2 \} \]

\[ = E \left\{ |\phi - \arcsin \left( \frac{\lambda_0}{2\pi d} \Delta z \right) |^2 \right\} \]  

(17)

where \( \hat{\phi} \) is the estimated angle from (15). It should be noted that the model (13) used to develop the algorithm, is based on the assumption that \( \Delta f^{(2i-1)} = \Delta f^{(2i)}, \forall i \in [1, M/2] \). Since this may not be perfectly true in reality, the differential operation in equation (6) does not remove completely the carrier frequency offset contribution. From these considerations, (13) can be written as

\[ z = \left( \frac{2}{M} \sum_{i=1}^{M/2} |\hat{\alpha}^{(i)}|^2 e^{2\pi (\Delta f^{(2i-1)} - \Delta f^{(2i)}) NT} \right) e^{j\psi} + W. \]  

(18)

If we can approximate the channel gains as \( \hat{\alpha}^{(i)} \approx \hat{\alpha}, \forall i \in [1, M/2] \), (13) becomes

\[ z \approx Ae^{j(\hat{\psi} + \epsilon)} + W \]  

(19)

where

\[ \epsilon = \frac{2}{M} \sum_{i=1}^{M/2} e^{2\pi (\Delta f^{(2i-1)} - \Delta f^{(2i)}) NT}. \]  

(20)

Then, computing the phase via the arctan(\( \cdot \)) function, for \( \Delta z \in \left[ -\frac{\pi}{2}, \frac{\pi}{2} \right] \), we obtain

\[ \Delta z = \arctan \left( \frac{\Re[z]}{\Im[z]} \right) \]

(21)

\[ \arctan \left( \frac{A \sin(\hat{\psi} + \epsilon) + \Re[W]}{A \cos(\hat{\psi} + \epsilon) + \Im[W]} \right) \]

where

\[ K(\hat{\psi}, \epsilon) = \frac{1 + \frac{\Im[W]}{\Re[W]}}{1 + \frac{\Re[W]}{A \cos(\hat{\psi} + \epsilon)}}. \]  

(22)

To proceed, we can write

\[ \tan(\hat{\psi} + \epsilon) \cdot K(\hat{\psi}, \epsilon) = \tan(\hat{\psi}_c) \]  

(23)

where \( \hat{\psi}_c = 2\pi f_{0,RF} \Delta t_c = 2\pi \frac{d}{\lambda_0} \sin(\phi_c) \) is the phase corrupted by the noise and the unperfect carrier frequency offset compensation. Therefore, we can express the MSE as

\[ MSE(\phi) = E \left\{ |\phi - \arcsin \left\{ \sin(\phi_c) \right\}|^2 \right\}. \]  

(24)

Averaging in time and space reduces the noise variance. However, in the presence of distinct carrier frequency offsets among the antennas, there exists a phase error \( \epsilon \) that depends on the averaging time period \( N \).

Ideally, the argument of \( z \) is

\[ \hat{\psi} = 2\pi \frac{d}{\lambda_0} \sin(\phi) \]  

(25)

because \( \frac{\cos \Delta t}{d} = \lambda_0 \). If we want to correctly estimate the DoA, the range of \( \psi \) has to be comprised between \( -\pi, \pi \) because of the periodicity of the \( \sin \) function. Hence, we can derive the constraint

\[ 2\pi \frac{d}{\lambda_0} |\sin(\phi)| \leq \pi. \]  

(26)

If we want to correctly resolve the widest possible DoA range (ideally \( \phi \in [-90^\circ, 90^\circ] \)), the constraint becomes

\[ d \leq \frac{\lambda_0}{2}. \]  

(27)

Note that the larger the distance \( d \), the larger the angle \( \hat{\psi} \) and, consequently, the lower the noise effect is.

To graphically show the observations above, we now consider the additive contribution on \( \psi \) caused by a DoA increment of \( 1^\circ \) (that is denoted as \( \Delta \psi(\phi, 1^\circ) \))

\[ \Delta \psi(\phi, 1^\circ) = \hat{\psi}(\phi) - \hat{\psi}(\phi - 1^\circ) \leq \pi |\sin(\phi) - \sin(\phi - 1^\circ)|. \]  

(28)
Gaussian components, we obtain
\[ d \text{sensors distance} \]
where the true. Neglecting constant additive terms, the logarithmic pdf
the estimator in the presence of noise.
\[ \Delta \hat{\psi} \]
the best attainable performance we evaluate the Cramer-Rao
unknown parameters vector
\[ \theta \]
- the th diagonal entry of the inverse of the Fisher
In Fig. 2 we plot the curves of \( \Delta \psi(\phi, 1^\circ) \) for three possible
distances \( d \). As we can see from the figure, the increment
\( \Delta \psi(\phi, 1^\circ) \) decreases for increasing angles \( |\phi| \) and for
decreasing antenna distances \( d \), which lowers the robustness of
the estimator in the presence of noise.

V. CRAMER-RAO BOUND

The MSE can be lower bounded with the Cramer-Rao
bound \([8],[9]\), i.e., \( MSE(\phi) \geq CRB(\phi) \). To benchmark
the best attainable performance we evaluate the Cramer-Rao
bound in the absence of DC offset and under identical carrier
frequency offsets for all antenna channels. By definition, the
CRB is the \( p \)-th diagonal entry of the inverse of the Fisher
information matrix \( F \), where \( p \) is the index of \( \phi \) in
the unknown parameters vector \( \theta = [A, \phi] \); the \((i, j)\)-th element
of the Fisher information matrix is given by
\[ F_{ij} = -E \left\{ \frac{\partial^2}{\partial \theta_i \partial \theta_j} \ln p(z(\theta)) \right\} \]
(29)
where \( p(z(\theta)) \) is the probability density function (pdf) of
\( z \) in (13), conditioned by the unknown parameters vector
\( \theta \). Assuming the noise \( W \) to have independent zero-mean
Gaussian components, we obtain
\[ p(z(\theta)) = \frac{1}{\pi \sigma^2_W} e^{-\frac{1}{\sigma^2_W} \left| z - Ae^{j\psi} \right|^2}. \]
(30)
We have verified via numerical fitting that Gaussianity holds
true. Neglecting constant additive terms, the logarithmic pdf
becomes
\[ L(\theta) = \frac{1}{\sigma^2_W} \left[ zAe^{-j\psi} + z^*Ae^{j\psi} - A^2 \right]. \]
(31)
Although not reported for space limitations, the Fisher
information matrix is diagonal so that the CRB is
\[ F_{pp}^{-1} = -\frac{1}{E} \left\{ \frac{\partial^2 L(\theta)}{\partial \phi^2} \right\} \]
(32)
with
\[
\frac{\partial^2 L(\theta)}{\partial \phi^2} = -\frac{1}{\sigma^2_W} \left[ \left( 2\pi \frac{d}{\lambda_0} \cos(\phi) \right)^2 \left( zAe^{-j\psi} + z^*Ae^{j\psi} \right) + \left( j2\pi \frac{d}{\lambda_0} \sin(\phi) \right) \left( zAe^{-j\psi} - z^*Ae^{j\psi} \right) \right].
\]
(33)
Finally, if the noise has zero mean, we obtain
\[ CRB(\phi) = \frac{1}{2 \frac{A^2}{\sigma^2_W} \left( 2\pi \frac{d}{\lambda_0} \cos(\phi) \right)^2}. \]
(34)
To proceed, we can write the noise variance as
\[ \sigma^2_W = E\{|W|^2\} \]
\[ = \frac{4}{NM^2} \sum_{i=1}^{M/2} \sigma^2_{\omega(i)} \]
\[ = \frac{4}{NM^2} \sum_{i=1}^{M/2} 2|\hat{\alpha}(i)|^2 \sigma^2_{\omega(i)} + (\sigma^2_{\omega(i)})^2. \]
(35)
Assuming the differential noise to be spatially and temporally
uncorrelated with variance identical for each index \( i \), i.e.,
\( \sigma^2_{\omega(i)} = \sigma^2_{\omega}, \forall i \in [1, M/2] \), we obtain the ratio
\[ A^2 = \frac{2}{M} \sum_{i=1}^{M/2} \frac{|\hat{\alpha}(i)|^2}{\sigma^2_{\omega}} \]
\[ = \frac{2}{M} \sum_{i=1}^{M/2} \frac{2|\hat{\alpha}(i)|^2}{\sigma^2_{\omega(i)}} + 1 \]
(36)
\[ = \frac{NM}{2} SNR_z^2 + \frac{1}{2 SNR_z^2} \]
where
\[ SNR_z = \frac{2}{M} \sum_{i=1}^{M/2} SNR_{z(i)} \]
(37)
with
\[ SNR_{z(i)} = \frac{|\hat{\alpha}(i)|^2}{\sigma^2_{\omega}} \]
\[ = \frac{SNR_{z(x,f)}^{(2i-1)} SNR_{z(x,f)}^{(2i)}}{SNR_{z(x,f)}^{(2i-1)} + SNR_{z(x,f)}^{(2i)}}, \quad i \in [1, M/2] \]
(38)
and
\[ SNC_{x(f)} = \frac{|\alpha(i)|^2}{\sum_{k=-N_h}^{N_h} |h_k|^2 \sigma^2_{\omega}}, \quad i \in [1, M]. \]
(39)
It should be noted that for high SNRs we have that
\( SNC_z \approx \frac{SNR_z}{2} \) and \( \sigma^2_W \approx \frac{NM}{4} SNC_{x(f)}. \) Hence, the use of
the proposed algorithm that performs a double differential
operation, reduces the SNR by a factor of 4 (6 dB). However, it
allows compensating the hardware impairments with a simple
method.
VI. SIMULATION RESULTS

The performance of the estimator has been evaluated also via simulations. We have assumed $N = 1$, $M = 4$, $f_0 = 3$ MHz, $f_c = 2.412$ GHz, and $d = \frac{\lambda}{2}$. The carrier frequency offsets for the antenna channels have been modeled as independent Gaussian random variables with mean value $m_{\Delta f} = 15$ kHz and standard deviation $\sigma_{\Delta f}$. The phase offsets $\Phi^{(i)}$ that model the effect of uncalibrated array have been considered as uniform random variables, with zero-mean and standard deviation $2\pi$. Finally, the DC offset has been well in the noise and treated as a constant interferer $w^{(i)}_{DC}$ yielding a given signal-to-interference ratio at the digital receiver filter input equal to $SIR^{(i)} = \frac{|a_{w^{(i)}_{DC}}|}{|w^{(i)}_{DC}|^2}$. The SNRs and SIRs have been assumed identical for the 4 antenna channels.

In Fig. 3, we report the MSE as function of the SNR, for different values of the SIR and $\sigma_{\Delta f}$. For $\sigma_{\Delta f} = 0$ Hz the carrier frequency offsets are identical for all antenna channels and they are equal to 15 kHz.

In the absence of a DC offset ($SIR = +\infty$) the CRB and the simulated performance are in excellent agreement for $\sigma_{\Delta f} = 0$ Hz. For increasing values of DC offset, the MSE curves exhibit an error floor and fall off the CRB (obtained under the assumption of null DC offset) as the SNR increases. In Fig. 4, we report the estimator MSE as function of the DoA, for different values of SIR and $\sigma_{\Delta f}$. Again, as the SIR increases the simulated curves approach the CRB for $\sigma_{\Delta f} = 0$ Hz.

The right plots in Fig. 3 and 4 show that when we assume the carrier frequency offset to be different across the antenna channels (with $\sigma_{\Delta f} = 10$ kHz), the estimator MSE is still close to the CRB if the DC offset is absent. With a DC offset, the error floors are a little more pronounced than those shown in the left plots because the distinct carrier frequency offsets cannot be fully compensated and a residual phase error $\phi$ exists.

In general, the DoA estimator performs well for a wide range of angles of arrival as Fig. 4. The most significant error is introduced by the DC offset that needs to be lowered with narrow band filtering. In the simulation we have used a conventional band-pass filter.

VII. CONCLUSION

We have presented an algorithm for DoA estimation in the presence of carrier/phase offsets and DC bias. Both the Cramer-Rao bound and simulation analysis show that the estimator is robust for a wide range of angles and SNRs, also in the presence of antenna channels that exhibit different carrier frequency offsets. The most significant cause of error is the presence of the DC offset introduced by the direct conversion RF front-ends. To lower the DC offset selective narrow band filtering is required.

REFERENCES