A Filtered Multitone Modulation Modem for Multiuser Power Line Communications with an Efficient Implementation

Andrea M. Tonello and Francesco Pecile

DIEGM - Università di Udine - Via delle Scienze 208 - 33100 Udine - Italy
phone: +39 0432 558042 - fax: +39 0432 558251 - e-mail: tonello@uniud.it, francesco.pecile@uniud.it

Abstract—In this paper we investigate filtered multitone modulation (FMT) for broadband multiuser powerline communications. FMT generalizes the popular OFDM scheme through the deployment of sub-channel shaping filters. We address the implementation problem, and we derive a novel efficient digital implementation of both the synthesis filterbank and the analysis filterbank assuming to multiplex the users via the tone allocation. A performance comparison with OFDMA is reported and shows that multiuser FMT has more robust performance in an asynchronous multiple access channel due to the higher spectral confinement of the sub-channels compared to OFDMA.

Index Terms—FMT modulation, frequency selective fading, multiuser systems, OFDMA.

I. INTRODUCTION

In this paper we consider filtered multitone modulation (FMT) for transmission over power line channels. FMT is a discrete time implementation of a multicarrier system where sub-carriers are uniformly spaced and the sub-channel pulses are identical (Fig. 1). Discrete Multitone Modulation (DMT) (also referred to as orthogonal frequency division multiplexing (OFDM)) can be viewed as an FMT scheme that deploys rectangular time domain filters [1]-[2]. FMT modulation has been proposed for transmission over broadband frequency selective channels both in very high speed digital subscriber lines (VDSL) [3], and more recently in wireless scenarios [4]-[5].

Broadband frequency selective channels introduce intercarrier (ICI) and intersymbol (ISI) interference at the receiver. The design of the sub-channel filters, and the choice of the sub-carrier spacing in an FMT system, aims at subdividing the spectrum in a number of sub-channels that do not overlap in the frequency domain, such that we can avoid the ICI and get low ISI contributions [3]. In an OFDM system the insertion of a cyclic prefix longer than the channel time dispersion is such that the ISI and ICI are eliminated, and the receiver simplifies into a simple one-tap equalizer per sub-channel. In FMT the sub-channel ISI is handled with sub-channel equalization. Provided that equalization is performed, FMT achieves higher spectral efficiency than OFDM because it does not require the cyclic prefix.

The FMT system can support user multiplexing in a FDMA fashion through the partition of the available tones across the users [4] similarly to OFDMA [6]. It is known that OFDMA suffers from multiple access interference (MAI) when the multiple access channel is asynchronous, i.e., the users signals are received with distinct propagation delays in excess of the cyclic prefix [7]-[8]. In this scenario, FMT has superior performance than OFDMA because of the sub-channel spectral containment that allows to maintain sub-channel orthogonality also in the presence of asynchronous users [4]-[5].

FMT has interesting properties that make it a good candidate for application also in power line channels, in particular:

- high spectral efficiency,
- capability of supporting multiuser transmission,
- robustness to frequency selective channels and to users asynchronism,
- sub-channel spectral containment that makes it robust to narrow band interference,
- possibility of shaping the spectrum by nulling undesired sub-channels.
However, the implementation of FMT can be more complex than OFDM because sub-channel filtering is required. An efficient polyphase implementation of the synthesis and analysis filter bank for single user FMT has been proposed in [3], and it is based on FFT and low rate filtering. Its complexity has been evaluated in [9].

In this paper we propose an alternative and novel implementation which is applicable both to the single user and the multiuser case. We look at the analysis filter bank and devise a polyphase architecture that allows to significantly lower complexity of the receiver in the presence of asynchronous multiple users.

A performance comparison in power line channels with OFDMA is reported, and it shows the superiority of FMT.

II. MULTIUSER FMT MODULATION SYSTEM

Notation is reported in Table I. We assume to have \( M \) available tones in the system. The users are frequency multiplexed through the assignment of a sub-set of the available tones.

The FMT signal transmitted by user \( u \) can be written as follows (Fig. 1)

\[
x^{(u)}(iT) = \sum_{k=K_u}^{K_u-1} \sum_{i=0}^{M_u-1} a^{(u,k)}(lT_0) g(iT-lT_0)e^{j2\pi f_k iT} \quad i \in \mathbb{Z}
\]

where \( a^{(u,k)}(lT_0) \) is the sequence of complex data symbols, e.g., M-QAM, that is transmitted on sub-channel \( k \) at rate \( 1/T_0 \), by user \( u \). \( K_u \subseteq \{0,...,M-1\} \) is the set of \( M_u = |K_u| \) tones assigned to user \( u \); \( M \) is the total number of sub-channels. \( T \) is the sampling period; \( W=1/T \) is the transmission bandwidth; \( T_0 = NT \) is the sub-channel symbol period; \( f_k = k/(MT) \) is the \( k \)-th sub-carrier; \( g(nT) \) is the prototype pulse; \( R = M/T_0 \) is the overall transmission rate in symbols.

If the sub-carrier spacing \( f_k - f_{k-1} \) is larger than \( 1/T_0 \) the scheme is referred to as non-critically sampled FMT, otherwise if \( f_k - f_{k-1} = 1/T_0 \) it is referred to as critically sampled FMT [3].

The implementation of the modulator according to (1) is inefficient. Assuming that the prototype pulse is FIR with \( L_g \) coefficients, it requires a number of complex operations (sums and multiplications) per output coefficient \( x^{(u)}(iT) \) equal to \( 2M_u\left\lfloor L_g/N \right\rfloor + M_u - 1 \).

A. Prototype Pulse

The prototype pulse can designed according to the guidelines in [9]. According to this design, the discrete time filter bank is with quasi-perfect reconstruction, i.e., with zero ICI and with small residual ISI. In Fig. 2 we plot the pulse impulse response, while in Fig. 3 we plot the frequency response for a number of sub-channels equal to \( M=16, 32, 64 \), and interpolation factor \( N=19, 35, 67 \). The aggregate transmission rate is respectively equal to \( M/T_0 = 16.8, 18.2, \)

19 Msymb/s assuming a transmission bandwidth of \( 1/T=20 \) MHz.

![Fig. 2. Prototype pulse for \( M/N=0.84, 0.91, 0.95 \).](image1)

![Fig. 3. Frequency response for \( M=16, 32, 64 \) corresponding to the pulses in Fig. 2.](image2)

B. Received Signal

The low pass signal (1) is digital-to-analog converted and transmitted over the communication channel. The received discrete time lowpass signal can be written as follows

\[
y(iT) = \sum_{u=0}^{N_u-1} y^{(u)}(iT - D^{(u)}) + \eta(iT)
\]

where
\[ y^{(u)}(iT) = \sum_{n} x^{(u)}(nT) g_{CH}^{(u)}(iT - nT) \]  

is the contribution due to the \( u \)-th user signal after propagation through the communication channel with impulse response \( g_{CH}^{(u)}(iT) \). \( D^{(u)} \) is the delay of user \( u \). \( \eta(iT) \) is the noise contribution.

C. Receiver Filter-Bank

In multiuser FMT the receiver can be implemented with a bank of single user receivers (Fig. 1). Each single user receiver compensates the propagation delay of the desired user, and runs a filter-bank (analysis filter bank) that is matched to the transmitter filter-bank. A property of FMT is that no ICI and MAI is present at the filter-bank output if the prototype pulse has bandwidth smaller than the sub-channel spacing. Each sub-channel, however, sees some ISI. The ICI contribution, the fourth term is the MAI, and the fifth second additive term is the ISI contribution, the third term is where the first term represents the useful data contribution, the error (MMSE) linear equalizer [10].

Use a simple one tap equalizer. In the simulation results that feedback equalization [3]-[4]. If the ISI is negligible, we can maximum likelihood sequence estimation, linear or decision. The ISI can be mitigated with some form of equalization, i.e.,

\[ \text{maximize } \sum_{n} x^{(u)}(nT) g_{CH}^{(u)}(iT - nT) \]

\[ \text{subject to } \sum_{k} \theta_{k}^{(u)} = 1 \]

III. EFFICIENT IMPLEMENTATION

Differently from OFDM, FMT requires sub-channel filtering. An efficient implementation of both the synthesis and the analysis filter banks has been proposed in [3]. The synthesis stage, comprises an M-point IDFT followed by low-rate filtering. The filters are obtained via the polyphase decomposition of the prototype pulse, and are cyclically time variant for non critically sampled FMT. The analysis stage is essentially the dual filter bank and requires low-rate filtering followed by an M-point DFT. We have studied the complexity of this implementation in [9]. In asynchronous multiuser FMT it should be observed that distinct analysis filter banks are required since the users have different propagation delays. That is, we require a polyphase filterbank for each user that is synchronized with one of the users time offset.

Herein, we propose an alternative and novel efficient implementation of both the synthesis and analysis filter banks. We start considering a user that deploys the tones in a generic manner. We assume frequency concentrated non-overlapping sub-channels the ICI and MAI terms are zero, such that

\[ z^{(u,k)}(mT_{0}) = a^{(u,k)}(mT_{0})\kappa_{EQ}^{(u,k)}(0) + I\!S\!I^{(u,k)}(mT_{0}) + I\!I\!S^{(u,k)}(mT_{0}) + \eta^{(u,k)}(mT_{0}) \]

where the first term represents the useful data contribution, the second additive term is the ISI contribution, the third term is the ICI contribution, the fourth term is the MAI, and the fifth term is the noise contribution. Further, \( \kappa_{EQ}^{(u,k)}(mT_{0}) \) is the equivalent sub-channel impulse response

\[ k_{EQ}^{(u,k)}(mT_{0}) = \sum_{i} g(iT) g_{CH}^{(u)}(nT - iT) e^{j2\pi(f_{k} - f_{n})} h(nT - mT_{0}). \]

If we assume frequency concentrated non-overlapping sub-channels the ICI and MAI terms are zero, such that

\[ z^{(u,k)}(mT_{0}) = a^{(u,k)}(mT_{0})\kappa_{EQ}^{(u,k)}(0) + I\!I\!S^{(u,k)}(mT_{0}) + \eta^{(u,k)}(mT_{0}) \]

The ISI can be mitigated with some form of equalization, i.e., maximum likelihood sequence estimation, linear or decision feedback equalization [3]-[4]. If the ISI is negligible, we can use a simple one tap equalizer. In the simulation results that we report in this paper we use a simple minimum mean square error (MMSE) linear equalizer [10].
\[ x_{(u,i)}(mT_2) = x^{(u)}(i + mT_2) \quad i = 0, \ldots, M_2 - 1, \quad m \in \mathbb{Z} \]
\[ = \sum_{i=0}^{M-1} \sum_{k=0}^{M_2-1} A_{(u,i)}(IT_0) e^{2\pi i \frac{m}{M} k} g(iT_2 + mT_2 - IT_0) \quad (8) \]
\[ = \sum_{i=0}^{M-1} A_{(u,i)}(IT_0) g^{(u,i)}(mL_2T_0 - IT_0) \]

where \( \{A_{(u,i)}(IT_0)\} \) are obtained by the M-point IDFT of \( \{a_{(u,i)}(IT_0)\} \) followed by a cyclic extension of \( M - M_2 \) elements (\( K \) times repetition), and
\[ a_{(u,i)}(IT_0) = \begin{cases} a^{(u,i)}(IT_0) & k \in K_u \\ 0 & \text{otherwise} \end{cases} \quad (9) \]

Further, the \( i \)-th polyphase component of the filter is
\[ g^{(u,i)}(mL_2T_0 - IT_0) = g((i + m)T_2 - IT_0), \quad i = 0, \ldots, M_2 - 1, \quad m \in \mathbb{Z}. \quad (10) \]

Therefore, the FMT signal of user \( u \) (Fig. 4) can be efficiently synthesized through an M-point IDFT, cyclic extension of the outputs, low-rate filtering with the pulses \( g^{(u,i)}(IT_0) = g((i + m)T_2), \) sampling with period \( L_2T_0 \), and P/S conversion.

If the tones are interleaved across the users, i.e., user \( u \) deploys the tones with index \( kN_u + u, \quad k = 0, \ldots, P - 1 \), the implementation can be simplified further since the block \( \{a^{(u,i)}(IT_0)\} \) is obtained by running an IDFT with \( P = M_u = M / N_u \) points, followed by a cyclic extension with \( M_2 - P \) coefficients, and a phase rotation. In formulae, \( \{a^{(u,i)}(IT_0)\} \) are obtained as
\[ A_{(u,i)}(IT_0) = e^{\frac{2\pi i}{M} \frac{M - M_2}{N_u} k} \sum_{k=0}^{M_2-1} a^{(u,i)}(IT_0) e^{\frac{2\pi i m}{M_2} k}, \quad i = 0, \ldots, M_2 - 1. \quad (11) \]

This implementation is shown in Fig. 5.

\[ B. \quad \text{Efficient Analysis Filter Bank} \]

The analysis filter bank for user \( u \) can be implemented via a polyphase decomposition of the received signal (Fig. 6), after compensation of the time offset, as follows
\[ y_{(u,i)}(mL_2T_0) = y((i + m)T_2 + D^{(u)}), \quad i = 0, \ldots, M_2 - 1, \quad m \in \mathbb{Z}. \quad (12) \]

Then, the filter bank output is computed as follows
\[ z^{(u,i)}(IT_0) = \sum_{i=0}^{M_2-1} Z_{(u,i)}(IT_0) e^{-\frac{2\pi i K_u k}{M_2}}, \quad k \in K_u \quad (13) \]
\[ Z_{(u,i)}(IT_0) = \sum_{m=0}^{M_2-1} y_{(u,i)}(mL_2T_0) h^{(u,i)}(mL_2T_0 - IT_0). \quad (14) \]

According to (14), the \( M_2 \) polyphase components of the input signal, are interpolated by a factor \( L_2 \) and analyzed with the low-rate filters \( h^{(u,i)}(IT_0) = h((mL_2T_0 - IT_0). \) Finally, application of a \( M_2 \)-point DFT yields the result. Only the outputs from the DFT of index \( K_u k, \quad k \in K_u \) are required and need to be taken into account.

An alternative implementation is obtained by making periodic, with period \( M_u \), the input block of size \( M_2 \) to the DFT. Then, after the periodic transform, an \( M \) point DFT is required to obtain the \( M \) outputs. The ones with index \( k \in K_u \) belong to the desired user \( u \). The periodic transform is implemented as follows
\[ Z^{(u,i)}_{(u,i)}(IT_0) = \sum_{n=0}^{K_u-1} Z_{(u,i+nM)}(IT_0), \quad i = 0, \ldots, M_2 - 1 \quad (15) \]

This implementation is shown in Fig. 7.

It should be noted that this receiver requires an analysis filter bank per user. This is because we assume that the users have different time delays and each filter bank needs to be synchronized with a different timing phase, one per user. On the contrary, if the users were synchronous, then a single analysis filter bank would be sufficient.

If the tones are interleaved across the users the analysis
filter bank of user \( u \) can be simplified further by deploying a \( P \)
point DFT on the block that is obtained by an appropriate de-
rotation and periodic transform, as follows
\[
Z_p^{(u,i)}(iT_0) = \sum_{n=0}^{M-1} e^{-j2\pi\frac{n+i}{M}} Z_n^{(u,i)}(iT_0),
\]
(16)
\[
Q = \frac{M_i}{P} = K_u N_u, \quad i = 0, \ldots, P-1
\]
This implementation is shown in Fig. 8.

IV. FRACTIONALLY SPACED ANALYSIS FILTER BANK
In the implementations above one filter bank per user is
required. With the interleaved tone allocation we have shown
that each of these filter banks requires a \( P \) point DFT. It would
be beneficial, for complexity purposes, to use a unique filter
bank at the output of which we simultaneously get the overall
\( M \) sub-channels for the \( N_u \) users. We propose to use only two
fractionally spaced analysis stages instead of \( N_u \) filter banks.
Each analysis stage is efficiently implemented as described in
Section III.B. The two analysis stages work with the sequence
of input samples
\[
y_o^{(i)}(mL_2T_0) = y(iT + mL_2T_0 + \Delta_o)
\]
(17)
\[
y_i^{(i)}(mL_2T_0) = y(iT + mL_2T_0 + \Delta_i)
\]
with \( \Delta_i = \Delta_o + T_0 / 2 \). The first sampling phase can be chosen
as \( \Delta_o = \langle \Delta_o \rangle + \min \langle \Delta_0 \rangle / 2 - T_0 / 4 \), \( \Delta_o = \text{rem}(D^{(o)}/T, N)T \). Then,
we can run fractionally spaced linear sub-channel
equalizers [10]. Note that with ideal band limited pulses
neither ICI nor MAI is present also with this receiver.

V. COMPLEXITY COMPARISON
In Table II we report several results about the evaluation
of complexity of the schemes herein derived. It is measured in
terms of number of complex operations (addition and
multiplications) per second. \( \alpha \) is a factor larger than 1 that
depends on the FFT implementation. A numerical comparison
is shown in Fig. 9 (assuming \( \alpha = 1 \)) for various sub-
channels and prototype pulse length in multiples of \( T_0 \).
In the left plot of Fig. 9 we consider only the complexity
of the analyses and synthesis filter banks. Herein we fix
\( M=32 \) for FMT, and a prototype pulse of length
\( L = \lfloor L_{g_{1,2}} / N \rfloor = 10, 32 \), while for OFDM the number of tones
is \( M=512, 1024 \). The tones are interleaved across the users.
OFDMA involves lower complexity than multiuser FMT. This
is due to the complexity introduced by sub-channel filtering
that increases linearly with the pulse length. However, as
shown in the next section FMT has superior performance.
Furthermore, herein complexity is studied in terms of complex
operations. However, it has to be said that what matters is
silicon area occupied in the hardware implementation (FPGA,
ASIC) such that the two solutions may require similar
hardware resources.

Furthermore, in the hardware implementation pre-filtering is
generally required before the DAC/ADC stages. Thus, in the
right plot of Fig. 9 we take into account also the complexity of
the pre-filters, and the one introduced by equalization (1 tap
for OFDM, and 5 taps for FMT). Since FMT has a more
compact spectrum than OFDM the low pass pre-filters are less

<table>
<thead>
<tr>
<th>Scheme Description</th>
<th>Number of operations per second</th>
</tr>
</thead>
<tbody>
<tr>
<td>OFDMA TX General</td>
<td>((\alpha M \log_2 M)(M + N_{CP}) / T)</td>
</tr>
<tr>
<td>OFDMA TX Interleaved</td>
<td>((\alpha P \log_2 P + M - 1)(M + N_{CP}) / T)</td>
</tr>
<tr>
<td>OFDMA RX General</td>
<td>((\alpha M \log_2 M)(M + N_{CP}) / T)</td>
</tr>
<tr>
<td>OFDMA RX Interleaved</td>
<td>((\alpha P \log_2 P + M - 1)(M + N_{CP}) / T)</td>
</tr>
<tr>
<td>FMT TX General</td>
<td>((\alpha M \log_2 M + 2N)(L_g / N - N) / T_0)</td>
</tr>
<tr>
<td>FMT TX Interleaved</td>
<td>((\alpha P \log_2 P + 2N)(L_g / N - N) / T_0)</td>
</tr>
<tr>
<td>FMT RX General</td>
<td>((\alpha M \log_2 M + M / (L_g / M - 1)) / T_0)</td>
</tr>
<tr>
<td>FMT Receiver Interleaved</td>
<td>((\alpha P \log_2 P + M / (L_g / M - 1)) / T_0)</td>
</tr>
</tbody>
</table>

Fig. 9. Complexity of multiuser FMT and OFDMA for the transmit filter bank
(solid) and receiver filter bank (dashed). Prototype pulse length \( L = L_{g_0} / N \).

Fig. 10. Performance comparison between multiuser FMT and OFDMA with 8
asynchronous users for different values of OFDMA tones.
uniformly distributed in $[0, T_0]$, with an interleaved tone allocation. The users’ time offset is prototype pulse ($\tau=\frac{T_0}{10}$) exhibits error floors as a result of the loss of orthogonality in $N_{CP}=48$ the channel is not fully compensated. Although not reported the performance that can be achieved with the fractionally spaced receiver for multiuser FMT is similar.

The results show that FMT is a more robust scheme than OFDM in the asynchronous multiple access channel.

VI. PERFORMANCE

In Fig. 10 we report a performance comparison between multiuser FMT and OFDMA assuming an asynchronous multiple access channel. The power line channel is generated according the statistical model in [11] whose realizations have an impulse response of duration equal to 4 ms. It is obtained from the Zimmermann-Dostert multipath model [12]. In the comparison we have considered the following system parameters. The overall bandwidth is fixed to $1/T=20$ MHz. $N_U=8$ asynchronous users are considered, and are multiplexed with an interleaved tone allocation. The users’ time offset is uniformly distributed in $[0, D_{\text{max}}]$, with $D_{\text{max}}=0$, $T_0$, $2T_0$, with $T_0=35$ $T$; 4-PSK signalling is used.

The FMT scheme deploys 32 tones, and the prototype pulse corresponds to the one in Fig. 2.B with $M=32$ and $N=35$. We consider also the performance achievable with the same pulse although truncated to $L=\left\lfloor \frac{L_{p,k}}{N} \right\rfloor = 10$ taps. A simple 5 taps linear MMSE equalizer is used to compensate for the sub-channel ISI. The OFDMA scheme deploys a number of tones equal to 512, or 1024, and correspondingly a CP length equal to $N_{CP}=48$, 96. Thus, the two schemes have the same aggregate data rate equal to 18.28 Msymb/s.

Fig. 10 shows that the BER performance of the FMT system is good, and basically insensitive to the users’ asynchronism both with the long ($L=32$) and the short prototype pulse ($L=10$). On the contrary, the OFDMA system exhibits error floors as a result of the loss of orthogonality in the presence of channel time dispersion and time offset in excess of the CP duration. We point out that the channel response has duration equal to $80T=4$ us. Therefore, for $N_{CP}=48$ the channel is not fully compensated. Although not reported the performance that can be achieved with the fractionally spaced receiver for multiuser FMT is similar.

The results show that FMT is a more robust scheme than OFDM in the asynchronous multiple access channel.

VII. CONCLUSIONS

We have investigated the use of FMT modulation for multiuser powerline communications. In particular, we have derived novel efficient implementations. Performance results show that FMT has better performance than OFDMA in typical powerline channels requiring a smaller number of sub-channels which allows to reduce the complexity of both the FMT transmitter and receiver. Undergoing work is concentrating in the analysing of the robustness of FMT to narrow band interference. Preliminary results show that it is superior to the one provided by OFDM because of the better sub-channel spectral containment.

REFERENCES