Extended Torque-Speed Region Sensor-Less Control of Interior Permanent Magnet Synchronous Motors

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Abstract - In this paper a sensor-less speed controller for an Interior Permanent Magnet synchronous motor is presented. It allows to fully exploit the torque-speed characteristic of the motor, also in the flux-weakening region. It is based on the estimation of a generalized back-EMF space vector by means of a Luenberger state and disturbance observer. The estimated rotor-magnet position and speed are then employed inside an optimised vector control scheme of direct and quadrature motor currents trajectories. Test results are presented, in order to verify the performance and demonstrate the effectiveness of the proposed solution.

I. INTRODUCTION

The so-called “Interior” Permanent Magnet (IPM) synchronous motor (build with magnets displaced inside the rotor body) represents an attractive solution for emerging applications, such as electric vehicles and domestic appliances.

Like the more popular “Surface” Permanent Magnet (SPM) synchronous motors (built with magnets displaced on the rotor surface), the IPM motors are characterized by the absence of rotor losses, that calls for “cool” rotor and increasing efficiency, and the high torque vs. weight ratio. Additional features are the robustness of the rotor structure, mechanically suited to high speed operation, and the presence of magnetic saliency, that is the “direct” $d$-axis inductance is substantially different from the “quadrature” $q$-axis inductance, where the $d$-axis is usually selected to be aligned with the PM flux axis according to the equivalent Park model of the synchronous machine. This characteristic is particularly suited for extending the torque/speed operating region by proper “field weakening” control techniques, [1]; and, also, it allows the application of some specific approaches to position and speed detection (“self-sensing” or “sensor-less” control), such as injection of high frequency voltage or currents investigating the rotor saliency, [2].

Despite of this, IPM motors are still far from a large diffusion, one of the reasons being certainly the difficult implementation of a controller able to fully exploit its peculiarities, a task strictly related to the specific motor design, [3]. In fact, the saliency gives rise to quite strong non-linear operating characteristics, often increased by saturation and mutual axes interaction (“cross-couplings”). Then, to take advantage of the motor features, the controller must fit the motor characteristics as close as possible and all over the operating range.

In this paper a sensor-less speed controller for an interior permanent magnet synchronous motor is presented. It allows to fully exploit the torque-speed characteristic of the motor, also in the flux-weakening region. It is based on the estimation of a generalized back-EMF space vector by means of a Luenberger state and disturbance observer, and allows an unified approach both for surface and interior PM motors. The estimated rotor-magnet position and speed are then employed inside an optimized vector control scheme of direct and quadrature motor currents trajectories. Test results are presented, using an experimental set-up based on the TMS 320F2812 Digital Signal Controller (DSC), in order to verify the performance and demonstrate the effectiveness of the proposed solution.

II. FICTITIOUS PERMANENT-MAGNET FLUX MODEL

An IPM synchronous motor having symmetrical wye-connected stator phases with isolated neutral connection and sinusoidal back-EMF is considered. Neglecting saturation and cross-couplings effects, the Park $dq$-model of the machine is:

$$
\begin{align*}
\psi_d &= L_d i_d + \omega \psi_q \\
\psi_q &= L_q i_q + \omega \psi_d \\
\psi_d &= L_d i_d + \hat{\psi}_M \\
\psi_q &= L_q i_q
\end{align*}
$$

(1)

where $\psi_{d,q}$, $i_{d,q}$, $\psi_{d,q}$ are the components of voltage, current and flux linkage of the equivalent $dq$-circuits, $L_{d,q}$ the respective synchronous inductances, $R$ the winding resistance, $\hat{\psi}_M$ the permanent magnet flux linkage, $\omega$ the rotor electrical speed, and $p = d/dt$. Motor electromagnetic torque can be calculated as:

$$
m_\tau = \frac{3}{2} PP [\psi_M i_q + \Delta L \cdot i_d]
$$

(2)

where $PP$ is the number of pole pairs, and $\Delta L = L_d - L_q$. 

By substituting fluxes expressions into voltage equations, the voltage model is obtained:

\[ v_d = R_s i_d + L_d p i_d + p \dot{\psi}_M - \omega L_q i_q \]
\[ v_q = R_s i_q + L_q p i_q + \omega L_p i_d + \omega \dot{\psi}_M \]

where synchronous inductances are assumed to be constant and permanent magnet flux linkage is considered as a time varying function. Previous equation can be mathematically manipulated in order to obtain an expression that is formally equivalent to that of a SPM, by defining the “fictitious” permanent-magnet flux as illustrated in Fig. 1, [4]:

\[ \dot{\psi}^\prime_M = \dot{\psi}_M + \Delta L \cdot i_d. \]

Permanent magnet flux linkage \( \dot{\psi}^\prime_M \) calculated from (4) can be substituted into (2) and (3) giving respectively:

\[ m_c = \frac{3}{2} p \dot{\psi}^\prime_M i_q \]
\[ v_d = R_s i_d + L_d p i_d + p \dot{\psi}^\prime_M - \omega L_q i_q \]
\[ v_q = R_s i_q + L_q p i_q + \omega L_p i_d + \omega \dot{\psi}^\prime_M \]

These relations show that IPM motor can be considered equivalent to SPM having \( L_q \) as synchronous inductance and the permanent magnet flux replaced by the fictitious one.

Equation (6) can be considered a generalised voltage model for both IPM and SPM, where proper significance of the permanent magnet flux linkage is assumed. If matrix form is considered and the transformation from the rotor-fixed \( dq \) to the stator-fixed \( \alpha \beta \) equivalent circuits is performed, the following relation is obtained:

\[ \begin{bmatrix} v_{\alpha} \\ v_{\beta} \end{bmatrix} = \begin{bmatrix} R \alpha & L_{\alpha q} \\ L_{\beta q} & R \beta \end{bmatrix} \begin{bmatrix} i_{\alpha} \\ i_{\beta} \end{bmatrix} + p \frac{\partial \psi}{\partial t} = \begin{bmatrix} R \alpha & L_{\alpha q} \\ L_{\beta q} & R \beta \end{bmatrix} \begin{bmatrix} i_{\alpha} \\ i_{\beta} \end{bmatrix} + p \frac{\partial \psi}{\partial t} \]

represents the space vector of the back-EMF related to the fictitious permanent magnet flux (in the following referred as “generalized” back-EMF), which coincides with the usual back-EMF when an isotropous motor is considered:

\[ \Delta L \rightarrow 0 : \quad v_{\alpha} \rightarrow p \dot{\psi}_M \frac{\cos \theta}{\sin \theta} \]

By assuming a steady-state condition for the direct axis current \( (i_d = 0) \), the components of the generalized back-EMF can be written:

\[ v_{\alpha} (\Theta) \cong -\omega (k_c + \Delta L i_d) \sin \theta \]
\[ v_{\beta} (\Theta) \cong \omega (k_c + \Delta L i_d) \cos \theta \]

being \( k_c \) the usual back-EMF constant. By this assumption the space vector of the generalized back-EMF is 90 degrees displaced with respect to the PM flux (see Fig. 1), and its amplitude

\[ v_i = \sqrt{v_{\alpha}^2 + v_{\beta}^2} = \left| \omega (k_c + \Delta L i_d) \right| \]

depends on motor speed and direct current.

Model (7) can be arranged into canonical state form:

\[ \begin{bmatrix} p \dot{v}_{\alpha} \\ p \dot{v}_{\beta} \end{bmatrix} = \begin{bmatrix} R_{\alpha} & L_{\alpha q} \\ L_{\beta q} & R_{\beta} \end{bmatrix} \begin{bmatrix} i_{\alpha} \\ i_{\beta} \end{bmatrix} + \begin{bmatrix} -\omega (k_c + \Delta L i_d) \sin \theta \\ \omega (k_c + \Delta L i_d) \cos \theta \end{bmatrix} \]

III. BACK-EMF OBSERVER

Model (11) can be manipulated aiming at the development of a dynamical state and disturbance observer for the estimation of the generalized back-EMF components (9), which contain in fact the rotor-magnet position information, needed for motor control. The generalized back-EMF disturbance can be modeled through a fictitious dynamics, such that:

\[ p \dot{v}_{\alpha} = 0 \]

Hereafter, from (11) and (13), the following 4th order dynamical system is obtained:

\[ \begin{bmatrix} p \dot{\chi} \\ p \dot{\nu} \end{bmatrix} = \begin{bmatrix} \bar{A} & \bar{B} \end{bmatrix} \begin{bmatrix} \chi \\ \nu \end{bmatrix} \]

where \( \chi = [i_{\alpha}, i_{\beta}, v_{\alpha}, v_{\beta}]^T \) is the state and disturbance vector, \( \nu = [v_{\alpha}, v_{\beta}]^T \) are the inputs and
are system parameters matrices. Model (15) is supposed to be linear and time-invariant. Hence, a Luenberger state observer can be build as follows:

\[
\dot{\hat{x}} = [A] \hat{x} + [B] \bar{u} + [K] (i - \hat{i})
\]

(16)

where \(\hat{x} = [i_\alpha, i_\beta, \dot{i}_\alpha, \dot{i}_\beta]^T\) is the vector of estimated quantities and \([K] \in \mathbb{R}^{4 \times 2}\) is a gain matrix. The Luenberger observer estimates the generalized back-EMF (and currents) components from the measurement of the motor phase voltages and currents; it uses the current estimation error (i.e. the difference between the measured and the estimated one) as correction feedback.

According to (9), upon the estimation of the back-EMF components, the magnet axis position is obtained by a reverse trigonometric formula such as:

\[
\hat{\theta} = \arccos \left( \frac{\hat{v}_\beta}{\sqrt{\hat{v}_{\alpha}^2 + \hat{v}_{\beta}^2}} \right)
\]

(17)

This approach leads to calculate the estimated speed as the derivative of the estimated position, Fig. 2.

**IV. EXTENDED TORQUE-SPEED CONTROL OF THE IPM MOTOR**

This section briefly recalls some hints about the torque-speed operation of an IPM, and how it can be optimized by proper current vector control strategy.

**A. IPM operation in the Park currents plane**

Neglecting the resistive drop at high speed operation, the square value of the steady-state voltage amplitude of the IPM is given by:

\[
v^2 = v_{\alpha}^2 + v_{\beta}^2 \geq \alpha^2 \left( \psi_{M}^2 + \psi_{q}^2 \right)
\]

(18)

Hereafter, the constant voltage (\(\bar{v}\)), torque (\(\bar{m}_e\)) and current (\(\bar{i}\)) loci in the Park currents plane can be found as follows:

\[
\left( \frac{\bar{\psi}}{\alpha} \right)^2 \geq \left( \psi_{M} i_d + \hat{\psi}_M \right)^2 + \left( \xi L_d i_d \right)^2
\]

(19)

**B. Optimum Control**

From the analysis of the characteristics curves recalled in the previous section, the trajectory of optimum operation of an IPM in the Park currents plane can be thought as represented in Fig. 4, [5]:

a) when the speed is below the rated value (“constant torque” region) there is no problem for the voltage limit, the operation
must lie over the maximum/torque/to/current ratio locus (maximum torque is achieved at point B); b) above the rated speed ("flux weakening" region), the operating zone is limited by the voltage limit ellipses (collapsing toward their center at increasing speed) and the rated current circle: maximum torque operation lies on the intersection of these curves (from point B to P); c) when the maximum torque/to/voltage ratio locus is attained (point P), maximum torque is possible over that locus (from point P to R); once the steady state speed is reached (in point R), operation lies on the voltage limit ellipse till to satisfy the required torque with minimum current (point Q).

Such description gives an idea of the complexity of the current control for an IPM. Differently from the SPM motor, where the \( d \)-current set point is always set to zero (and the \( q \)-current one is simply proportional to the torque), in case of the IPM both the \( dq \) currents need to be commanded according to the torque/speed operating point.

![Fig. 4 – Optimum trajectory and operating points in the Park currents plane.](image)

### C. Implementation

The optimum control strategy can be implemented through three "vector-control characteristics" (\( F_t, F_d, F_q \)) according to the scheme shown in Fig. 5. These characteristics will provide the set points of the \( dq \) currents (\( i_q^*, i_d^* \)) as functions of the torque command (\( m_c^* \)) and the operating speed, that is \( i_q^* = F_q(m_c^*,\omega) \), \( i_d^* = F_d(m_c^*,\omega) \), where the value of the speed will affect the control characteristics in the flux weakening region only. Moreover, in order to assure that the commanded operating point is attainable, a proper limitation of the torque reference with the speed is necessary, which corresponds to the maximum torque vs. speed curve achieved by optimum control, \( m_{clin} = F_t(\omega) \).

The back-EMF observer provides estimated rotor-magnet position (\( \hat{\theta} \)) and speed (\( \hat{\omega} \)) for current vector control, speed control loop, and current trajectories generation. In order to simplify the implementation, the command values of the feeding voltages (\( v_{a*}^*, v_{b*}^* \)), known by the controller, are employed instead of their measurements.

![Fig. 5 – Proposed sensor-less vector control scheme for the IPM.](image)

### V. EXPERIMENTAL RESULTS

The experimental system is shown in Fig. 6. It includes the IPM motor and the respective brake, a general purpose voltage source inverter (equipped with IGBT power switches), a control module with embedded TMS 320F2812 DSP. An host PC, linked to the control module by the serial interface, is employed both to run the software development environment and for real-time setting of control parameters and set-points. The control module includes a DAC converter, which allows real-time oscilloscope display of the variables computed by the digital controller. The inverter is operated at 10kHz PWM frequency. Main data of

<table>
<thead>
<tr>
<th>IPM MOTOR RATED VALUES AND PARAMETERS</th>
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<tbody>
<tr>
<td>Number of poles pairs</td>
</tr>
<tr>
<td>Speed (rpm)</td>
</tr>
<tr>
<td>Torque (Nm)</td>
</tr>
<tr>
<td>Current (A)</td>
</tr>
<tr>
<td>Ld (mH)</td>
</tr>
<tr>
<td>Lq (mH)</td>
</tr>
<tr>
<td>Rs (ohm)</td>
</tr>
<tr>
<td>back-EMF constant (V/Krpm)</td>
</tr>
</tbody>
</table>

![Fig. 6 – Experimental system.](image)
the IPM motor are reported in Table I. Both steady state and transient operation are investigated in order to verify the performance of the proposed sensor-less approach both in the constant torque and flux-weakening regions. Results are presented in “per units” values, assuming the rated speed/current/voltage as base values, and $2\pi$ as base position.

Fig. 7 shows a motor start-up with the target speed located at 3000 rpm in the flux weakening region, with 1.5 Nm load torque. According to the optimum control strategy, the transient $dq$ current are generated to accelerate at maximum torque capability both in the constant torque (moving on the maximum/torque/current ratio curve) and in the flux weakening region (moving on the rated current circle). The sensor-less algorithm is operated off-line in order to evaluate the convergence proprieties. The estimated position converges to the actual one inside the first electrical period, and maintains a good tracking over the whole test. The peak value of the position estimation error ($\Delta \theta = \hat{\theta} - \theta$) is limited to few degrees, it is slightly larger during the transient period, and reduces when steady-state is reached. The strong variations of the $d$ current, following optimal trajectory during transient, do not affect the position estimation; hence, the assumption of $pi_d = 0$ in computation of the generalized back-EMF observer, can be accepted.

Following figures refer to sensor-less operation at different steady-state conditions. Fig. 9 shows the estimated and actual position at 3000 rpm, 1.5 Nm. The observer confirms the good
performance at high-speed operation, as expected for a back-EMF based approach. Fig. 9 shows the estimated and actual $\alpha\beta$ currents, at medium speed (1210 rpm), 1.5 Nm, demonstrating the good behaviour in term of current observation, obtained by proper setting of the respective gain matrix. Fig. 10 refers to low speed operation at 100 rpm, 1.5 Nm. As expected by a back-EMF based approach, the performance reduces due to the lack of accuracy in voltage inputs and parameter discrepancies: the estimated $\alpha\beta$ components become distorted, and the position estimation deteriorates accordingly.

Finally, Table II shows some indicative values of the position estimation error at different speed and loading conditions, computed as the average of the peak values over the cycle. Experiments demonstrate that the position error is strongly (but not linearly) affected by the observer gains. Hence, in order to achieve minimum error, gain values are adjusted by a proper mapping according to the torque/speed operating conditions.

### VI. Conclusion

In this paper a sensor-less speed control of an interior permanent magnet synchronous motor is presented. The method is based on the estimation of a generalized back-EMF space vector by means of a Luenberger state and disturbance observer, and allows an unified approach both for surface and interior PM motors. The sensor-less technique can be implemented in software, it requires the knowledge of the motor currents and voltages only, and allows implementation through a standard digital signal controller and the same hardware needed for usual vector control.

Experimental tests confirm the effectiveness of the proposed solution in a large speed range, including flux weakening, and its compatibility with optimum torque-speed control strategies. In case of very-low speed (less than 5 Hz), or stand-still operation, the method looses its performance: some specific procedures, such as injection of high frequency voltages or currents investigating the rotor saliency, can be used to overcome the problem.

### Table II

<table>
<thead>
<tr>
<th>Speed (rpm)</th>
<th>No-load</th>
<th>1.5 Nm</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>32°</td>
<td>30°</td>
</tr>
<tr>
<td>500</td>
<td>16°</td>
<td>13°</td>
</tr>
<tr>
<td>1000</td>
<td>11°</td>
<td>14°</td>
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<td>1500</td>
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<td>9°</td>
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</tr>
<tr>
<td>2500</td>
<td>3°</td>
<td>6°</td>
</tr>
<tr>
<td>3000</td>
<td>2°</td>
<td>5°</td>
</tr>
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### References


